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Mechanics and physics of gas bubbles in liquids: a report on Euromech 98

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1. Introduction

Euromech 98 was held in Eindhoven in November 1977 with the authors as chairmen. The Colloquium was attended by 48 participants from a number of European countries.

Bubbles play an important role in many areas of technology: propeller-induced cavitation in ship building, cavitation in fluid machinery, nucleate boiling in reactors and similar devices, and many processes (centrifuges, mixers) in the chemical process industry. A wide variety of research, both of fundamental and of more applied nature, is going on in universities and in industrial laboratories. Representatives from all the above areas were included among the participants and consequently the topics raised in the presentations and discussions covered a very broad field.

The scene was set to some extent in the general lecture on the dynamics of cavitation bubbles and vapour bubbles by Prosperetti* (University of Milan)†. As always when specialists in boiling and in cavitation meet, the differences between these two processes tend to become hard to see (Prof. Bonnin commented in a lively way on this later in the Colloquium). Still there is, roughly speaking, the difference that at least most of the time growth and collapse of vapour bubbles is controlled by heat transfer whereas in cavitation the dynamics are governed by inertia effects. The processes in cold liquids (cavitation) are mathematically governed, for spherical bubbles, by the equation

$$\rho\left\{R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2\right\}=p_i-p_\infty-\frac{2\sigma}{R}-\frac{4\mu}{R}\frac{dR}{dt}, \tag{1}$$

which equation is therefore central in the subject. Here ρ is the liquid density, R the bubble radius, p_i and p_∞ the pressures inside the bubble and at large distance respectively, σ the surface tension, μ the viscosity of the liquid and t the time. Equation (1) is known as the Rayleigh–Plesset equation.‡ In addition to reviewing the analytic and numerical work done on this equation as presented recently in Plesset & Prosperetti (1977), Prosperetti discussed in some detail his own work on steady and transient nonlinear oscillations of gas bubbles in liquids (Prosperetti 1975, 1976). One of the outcomes of this work is a possible explanation for the $\frac{1}{2}$-subharmonic peak in the spectrum of cavitating bubbles, see Haussmann & Lauterborn*. While (1) governs

† The papers marked with an asterisk were presented at the Colloquium.

‡ Also sometimes called the RPNNP equation, standing for Rayleigh–Plesset–Noltingk–Neppiras–Poritsky. One cannot help but be reminded here of the WKB or WKBJ etc. method in applied mathematics.

spherically symmetric growth and collapse, of great importance also is the study of non-spherical bubble dynamics, the most important example being the collapse of a cavitation bubble near a solid wall. A striking phenomenon here is the development of a micro-jet directed towards the wall and responsible for cavitation damage. Numerical calculations (Plesset & Chapman 1971) can reproduce the initial formation of the jet and are confirmed by experiments (Lauterborn & Bolle 1975) with laser-produced cavitation bubbles discussed during the Colloquium by Lauterborn*.

For bubble behaviour in hot liquids (boiling), the central theory is that by Plesset & Zwick (1954). The growth of a bubble is limited by heat transfer to the bubble which is assumed to take place in a narrow layer around the growing bubble. In the Plesset–Zwick theory this results in the following integral equation for the variable

$$p = (R/R_0)^3$$

in terms of u:

$$p^{\frac{2}{3}}\frac{d^2p}{du^2}+\tfrac{7}{6}p^{\frac{4}{3}}\left(\frac{dp}{du}\right)^2 = 3\left[1-\mu\int_0^u (u-v)^{-\frac{1}{2}}\frac{dp}{dv}(v)\,dv-\frac{2\sigma}{p^{\frac{1}{3}}}\right], \tag{2}$$

where

$$u(t) = \frac{\alpha}{R^4}\int_0^t R^4(\theta)\,d\theta.$$

R and σ have been defined before, μ and α are complicated parameters composed of material properties and initial data. Prosperetti discussed well-known asymptotic solutions of (2) such as

$$\dot{R} \sim \left(\frac{2}{3}\frac{p_0-p_\infty}{\rho}\right)^{\frac{1}{2}}, \tag{3}$$

representing inertial growth, and

$$\dot{R} \sim \frac{1}{(Dt)^{\frac{1}{2}}}\left(\frac{T_\infty-T_b}{\rho_v^e L}\right)\lambda, \tag{4}$$

representing thermal growth at large times. In the above expression, D is the heat diffusivity, λ the heat conductivity of the liquid, ρ_v the vapour density corresponding to boiling temperature and L the latent heat. $T_\infty - T_b$ is the superheat. A complete numerical solution of (2) has not been carried out thus far but Prosperetti promised numerical results (Prosperetti & Plesset 1978) in the near future. He emphasized that here also, as in cavitation, there is a need for work on non-spherical bubbles.

2. Vapour bubbles in boiling

A number of contributions to the Colloquium were directly concerned with the growth of bubbles in super-heated liquid or their collapse in sub-cooled liquids. Loader* (Reading University) posed the following question in his contribution: how can we predict the behaviour of vapour-bubble collapse in sodium by doing experiments in water? He suggested that this could be done by taking the parameter $\mathscr{L}$, defined by

$$\mathscr{L} = \frac{\rho_v^e L}{(\lambda\rho c)^{\frac{1}{2}}}\frac{1}{T_0}\frac{R_0}{t_0^{\frac{1}{2}}}, \tag{5}$$

the same in sodium and water. Apart from quantities already mentioned, we have

$$T_0 = p_e\left(\frac{dT}{dp}\right)_{\text{saturation}}, \tag{6}$$

$$t_0 = R_0\rho/(p_\infty - p_v). \tag{7}$$

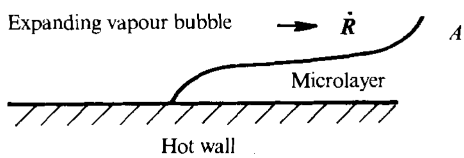

FIGURE 1. Illustrating the analogy made by Zijl & Van Stralen between growth of a vapour bubble and withdrawal from a heavy liquid.

Of course one parameter like (5) cannot be expected to describe the complicated events adequately, but, according to Loader, equation (5) used as a similarity rule produces at least an upper bound for the pressure at which an experiment in water should be carried out in order to predict behaviour in sodium. Jones* (Euratom, Ispra) talked about a mathematical model for growth or collapse of vapour bubbles in which mass diffusion is also allowed for. In his calculations Kármán–Pohlhausen types of approximations were used for the temperature and concentration profiles in the boundary layer around the bubble. Other approximations were introduced too and because of this the validity of the theory is hard to assess. Bonnin* (Electricité de France, Chatou) presented an asymptotic theory for growing bubbles also allowing mass transfer. His calculations were restricted to the regime where similarity solutions of equations like (2) are valid, but which were extended to include mass diffusion.

A number of presentations were concerned with bubble growth in the vicinity of a rigid wall which is, of course, an important practical situation. Anderson* (Imperial College, London) described how he tried to get some insight into the phenomenon of bubble growth by simulating it with the growth of a two-dimensional air bubble. The idea behind this is that the vapour-bubble growth is asymptotically like $t^{\frac{1}{2}}$, see (4). The same growth rate is obtained if a two-dimensional air bubble is injected with air at a constant volume rate. The experiment was done by blowing air through the bottom of a liquid-filled slot between parallel vertical walls. The fluid behaviour was observed near the expanding top of the bubble and near the collapsing base. Interesting as the experiment may be in itself, the relation with the boiling problem remains a little bit unspecified.

Zijl & Van Stralen* (University of Technology, Eindhoven) have set up a numerical program for the growth of a vapour bubble at a hot wall. No spherical shape is assumed (as in the Plesset–Zwick theory), and the shape of the bubble and associated motion in the liquid are expressed in terms of Legendre polynomials. An interesting point, which aroused quite a discussion, is the way Zijl & Van Stralen deal with the microlayer between the hot wall and the vapour bubble (see figure 1). Viewed by an observer moving with the expanding bubble, the wall moves with velocity $-\dot{R}$ to the left and there is an inertia force proportional to $-\ddot{R}$. Zijl & Van Stralen compare this with the withdrawal of a wall with velocity $-\dot{R}$ out of a heavy liquid. For this case there is an approximate theory by Landau and Levich (Levich 1962) for liquids with large surface tension, in which case the shape near A is approximately the same as that of the static meniscus. Van Stralen (Eindhoven) showed a movie about growth of vapour bubbles in super-heated liquids, which was especially informative for those participants more familiar with bubble dynamics in cold liquids.

A very interesting experiment was reported by Cooper* (University of Cambridge). Most theories, like the Plesset–Zwick theory, ignore gravity because it complicates calculations very much. Experimentally one should conduct experiments also under the most simple conditions. Therefore Cooper and co-workers built an ingenious free-falling test chamber in which the growth of vapour bubbles at a wall could be observed under zero-gravity conditions. Results of observations show that the changes of shape of those bubbles can be described in terms of a relatively simple parameter.

Summarizing this section on vapour bubbles we can say that a lot is known about spherical bubbles, and attention is now focused on such items as: growth of bubbles near a wall, the microlayer near the base of such bubbles, numerical studies. In general it can be said that there is a need for reliable experimental observations.

3. Optically and acoustically generated cavitation

Lauterborn* (University of Göttingen) summarized in his invited review lecture, for the particular benefit of those participants not working in cavitation, general features and characteristic aspects of cavitation and cavitation bubbles. He went on to review the work by himself and his group in Göttingen on optical cavitation. Cavitation bubbles are generated in liquids by giant laser pulses (0·15 joule laser output in 30–50 ns).

High stresses and temperatures are generated locally and cavitation bubbles are formed by physical processes not quite understood. The advantage of this with respect to flow-induced cavitation is that spherical bubbles growing and collapsing under controlled conditions can be observed in this way. Observations are made with rotating-mirror cameras. Lauterborn showed movie pictures of growing and collapsing bubbles. These pictures allow detailed verification of solutions of (1). In order to verify the results predicted by the viscous term in (1), measurements were done in very viscous liquids like silicon oil. Fascinating pictures were shown of the formation of a microjet when a cavitation bubble collapses near a wall. From these observations, as mentioned in §1, the numerical results by Chapman & Plesset could be verified. These computations describe the formation of the jet but break down when the jet impinges on the opposite part of the bubble wall. The real process, of course, goes on but it turned out that the formation of the microjet takes place so rapidly that even a speed of 3×10^5 frames/second is not sufficient to record accurately what happens. Other phenomena shown were interactions between collapsing bubbles with and without the presence of a wall.

Apart from Lauterborn's review lecture, a number of other contributions came from Lauterborn and his associates. Ebeling & Lauterborn* showed results of cinematic holography concerning shock waves emanating from collapsing cavitation bubbles generated with laser pulses. When comparing the observed behaviour of the bubble radius with theory, there is a difficulty that the experimental equilibrium radius is unknown. In Ebeling & Lauterborn's work the effective equilibrium radius was calculated by assuming that damping is due to energy radiated away with the shock wave. They used Gilmore's (1952) theory for this. However, during the discussion Prosperetti argued that the energy carried away by the shock wave is very small, and that the real process responsible for the observed strong decrease of the radius is condensation.

Hinsch & Brinkmeyer* (University of Göttingen) have studied the duration of a shock wave coming from a cavitation bubble during the final stage of collapse. In the

1950s this was estimated to be of order of 10^{-5}s. Taking advantage of the properties of laser light, the time resolution could be extended to 2 ns and the shock duration (passage of a shock at given location) was found to be of the order of 10 ns, the corresponding spatial width then being of the order of 60 μm. Also, pressures were measured during collapse and found to be as high as 50 bar at a distance of 1 mm from the cavitation centre. Haussmann & Lauterborn's* contribution was concerned with the noise spectrum of acoustically generated cavitation. They found, as others have before, a peak in the noise spectrum at one half of the driving frequency. In this contribution they showed that at the same time as this peak appears, strong ultra-harmonic components (that is to say at frequencies $\frac{1}{2}n$ ($n > 1$) times the driving frequency) also set in. Haussmann & Lauterborn suggest that ultra-harmonic bubble resonances are responsible for the appearance of the spectral subharmonic (Lauterborn 1976). Prosperetti raised in the discussion the point that it remains to be clarified how the subharmonic is associated with inception of cavitation, as observed experimentally. His hypothesis is that the subharmonic is excited by transient pressure perturbations caused by shock waves radiated from collapsing bubbles in the vicinity (Prosperetti 1975).

4. Flow-induced cavitation

Van der Meulen* (Netherlands Ship Model Basin, Wageningen) discussed in some detail the quite interesting relation observed for axisymmetric bodies between the onset of cavitation and the occurrence of a laminar separation bubble (Van der Meulen 1976; Arakeri 1973). He went on to show by way of pictures taken from holograms that for other types of boundary layers (with no laminar separation bubble) a correlation exists between boundary-layer structure and cavitation characteristics. Oldenziel* (Hydraulic Laboratories, Delft) was concerned with cavitation in fluid machinery, in particular behind valves in a water circuit.

As an important parameter Oldenziel introduces the cavitation susceptibility defined as follows. A sample of the employed fluid (mostly water) is led through a narrow venturi tube; in the throat of this tube exploding cavitation bubbles are registered with a lamp–photodiode system. The number of exploding bubbles registered per unit time as a function of the pressure in the throat is defined as cavitation susceptibility. With water for which bubble content and cavitation susceptibility are independently determined, the cavitation intensity behind butterfly valves has been measured. Oldenziel discussed the observed cavitation characteristics and compared the observations with predictions resulting from bubble dynamics.

Noordzij* (Netherlands Ship Model Basin, Wageningen) drew attention to the large pressure variations on the hull of a ship caused by unsteady cavitation on the propeller blades turning through the non-uniform wake behind the ship. Measurements show that the pressure amplitudes may be ten times larger than they are under non-cavitating but otherwise identical circumstances. Noordzij showed a high-speed movie taken in the NSMB's vacuum tank, on which the unsteady cavitation pattern could be clearly observed. The point of this recently built facility is that by adjustment of the 'atmospheric pressure' the cavitation number for the scale model can be made the same as that for full-size operation conditions. Rees* (Atomic Weapons Research Establishment, Foulness) showed a movie of an underwater shock wave reflecting off a free surface. The movie is a byproduct of research conducted ten years ago in reactor safety. When an explosion occurs, a shock wave may develop under certain conditions in the coolant. At the time the primary interest was in the reflexion of such a shock against the free surface of the coolant and the subsequent motion of the free surface. The results are also of interest for the study of cavitation. One could observe very clearly how the expansion wave which is formed at reflexion generates cavitation. The cavitating zone propagates from the surface downward into the interior of the fluid. The free surface itself moves upward and exhibits instabilities of the Rayleigh–Taylor type shortly after the arrival of the pressure waves.

5. Collective phenomena

A number of contributions were concerned with the way in which aspects of the mechanics and physics of single bubbles appear when one is dealing with large numbers of bubbles. Mørch* (Danmarks Tekniske Højskole, Copenhagen) considered the concerted collapse of a hemi-spherical cloud of cavitation bubbles above a solid wall. In Mørch's theoretical model the collapse starts in an outer shell of the hemisphere and spreads inward. Thereby a fraction γ of the energy of the cavity is supposed to be transferred to the next shell and so on. High pressures were obtained numerically at the wall when it was assumed that a large fraction of the energy in one shell is absorbed by the next. In the discussion van Wijngaarden mentioned his own (van Wijngaarden 1964) work on a similar problem. In that work the collapse propagates through the liquid because cavity collapse in one shell sets up fluid motion which by the associated pressure gradients affects the collapse in the next shell. In case of cavitation on a hydrofoil the pressure field at the wall may have maxima as high as 100 bar. In Mørch's calculations pressures of this magnitude were found also. They depend however on the assumed value of the transfer coefficient γ.

Persson* (Norske Veritas, Oslo), dealing in his contribution to the Colloquium with a similar problem, opted for a discrete attack and wrote down the velocity potential in the liquid as the sum of potentials of the type

$$\phi_j \sim R^2\dot{R}/|\mathbf{r}-\mathbf{r}_j|,$$

each of which represents the motion due to a collapsing cavity. Upon calculating the average pressure in the mixture with the help of Bernoulli's theorem one meets the difficulty, familiar in this type of problem, of divergent integrals when one integrates over all possible $|\mathbf{r}-\mathbf{r}_j|$. Persson circumvented this difficulty by restricting the integration to a finite volume. In this way he could obtain estimates for pressures on a hydrofoil during cavitation. Because of the cut-off in the integration, the significance of the results is hard to estimate. Similar objections were made with respect to the calculations by Tilmann* (Fraunhofer Gesellschaft, Ottobrun) of the viscosity of a suspension of small spheres in a Newtonian liquid. For a dilute suspension difficulties with non-convergent integrals can be overcome by using Batchelor's technique (see e.g. Batchelor 1974). So Batchelor & Green (1972) calculated the shear viscosity of a suspension up to $O(c^2)$, c being the volume concentration of spheres in a Newtonian liquid. Tilmann dealt with the same problem using, however, a so-called cell method (see e.g. Happel & Brenner 1965), the accuracy of which is hard to estimate. It appeared during the discussion that the order-c^2 term of Tilmann differs from the corresponding one found by Batchelor & Green, which is exact to this order. Tilmann reported, in another contribution to the Colloquium, a theoretical study regarding a suspension of small spheres in an incompressible inviscid liquid, paying attention primarily to relative motion. One of his simplifications, apart from using cell-methods as in the previously mentioned study, is to neglect the local fluctuations in the velocity at a point in the liquid caused by the changing positions of spheres in its vicinity. His result for the added mass as a function of concentration c differs in the first-order term from the result, exact in this order, obtained by van Wijngaarden (1976).

Van Oort* (Twente University of Technology, Enschede) reported on work in progress on expansion waves in suspensions of small gas bubbles in water. The reason for studying expansion waves, apart from intrinsic interest, is that the assumption of a nearly spherical shape is better justified for an expansion wave than for a compression wave, because the spherical expansion of a gas bubble in water is stable, but the collapse unstable. Van Oort started by reviewing the theory in van Wijngaarden (1968). Subsequently he made use of this theory to obtain analytic solutions for the pressure variations in the outskirts of an expansion wave. Experimental results show qualitative agreement with these solutions but the need for more controlled conditions in the experiments was stressed.

6. Effects of viscosity, surface tension, etc.

The effects of viscosity, surface tension and other properties were discussed in the contributions mentioned in this section. Coutanceau & Thizon* (University of Poitiers) gave results of a numerical computation of the resistance experienced by a spherical bubble rising in a liquid contained in a cylindrical pipe. The continuity of velocity and stress at the gas–liquid interface is satisfied. The Stokes approximation to the equations of motion is used. The analysis contains two parameters, viz. (sphere radius)/(pipe radius) = λ and (liquid viscosity)/(gas viscosity) = ω. Results were presented of the velocity of rise of a bubble as a function of these parameters. For $\omega \to 0$ (rigid spheres) the results agree with known ones (Happel & Brenner 1965). In the second part of their contribution, Coutanceau & Thizon showed experimental results for the shape of a bubble rising in a water-filled tube. It appeared that a spherical shape is maintained for $\lambda \lesssim 0{\cdot}2$. At values of λ between 0·2 and 0·6 the bubble is ellipsoidal (figure 2, plate 1). For $\lambda \sim 0{\cdot}98$ the bubble is cigar shaped with spherical caps. Finally a picture was shown of a bubble rising in a non-Newtonian liquid (figure 3, plate 2). A striking feature of the shape compared with the ellipsoidal shape seen in Newtonian liquids, like water, is the fact that the downstream end is sharp, unlike the upstream, end which is blunt.

A similar observation was made by Hassager* (Danmarks Tekniske Højskole, Copenhagen), whose primary aim was to investigate normal-stress effects on the flow along a bubble. He calculated the shape theoretically. Experiments carried out with the intention of verifying the theory were not conclusive because it appeared that the wall effect in the tube in which the experiments were done was larger than expected.

A picture of a bubble rising in a solution of polyacrylamide in glycerine had a remarkable resemblance to that shown by Coutanceau & Thizon. Feuillebois & Lasek* (Laboratoire d'Aérothermique C.N.R.S , Meudon) presented a second-order boundary-layer calculation for the unsteady boundary layer formed on a spherical bubble when the outer flow is strongly accelerated. It is assumed that owing to surface-active agents the no-slip condition applies on the surface of the sphere. During the discussion it appeared that their result for the force on the sphere is identical with that given by Landau & Lifshitz (1959, p. 97), who completely neglected the convective acceleration term but did not introduce the boundary-layer approximation. This can be understood by noting that when the fluid is accelerated from rest, at small times the local acceleration dominates and that viscous effects are confined to a thin boundary layer around the sphere. Johann* (Technische Hochschule, Darmstadt) made a study of the effect of a surface film on the motion of a gas bubble. He discussed the motion of a gas bubble covered by a film of stearic acid when it rises in a liquid confined between two nearly horizontal flat plates (Hele-Shaw cell). The steady motion of such a circular bubble has been studied by Johann both theoretically and experimentally. Some of the predictions made by the theory such as the motion in the liquid near the bubble, were borne out quite satisfactorily in the experiments.

7. Miscellaneous

In this section those contributions to the Colloquium are summarized which cannot be categorized in one of the previous sections. Chesters* (University of Technology, Delft) discussed the observed 'dimpling' of the surfaces of two gas bubbles when coalescing in a liquid. In Chesters's contribution a calculation was presented of the flow characteristics in the liquid film between the bubbles during coalescence. From the results of this calculation the observed dimpling could be explained. Chesters dealt also with the question whether in the final stage of coalescence van der Waals forces have to be included in the analysis. These ought to be included when the distance is of the order of 1000 Å. Since the time to reduce this distance to zero is very short, 10^{-4}s, on the basis of the final velocity calculated without van der Waals forces, it turns out that inclusion of van der Waals forces would not make much difference in the time of coalescence.

Lapicoré *et al.** (Centre d'Etudes Nucléaires de Cadarache, Saint Paul Lez Durance) presented work on spherical-cap bubbles. In the first part experiments to generate spherical cap bubbles were reported by Lapicoré. An ingenious apparatus consisting of quarter-spheres to pre-form air pockets in the shape of spherical caps was discussed. In the second part Berna outlined a theory to calculate flows containing a closed free surface like a large gas bubble. A functional is formulated which has to be minimized. Chincholle* (University of Paris) talked about his favourite 'effet fusée', that is to say the effect of increasing fluid momentum which owing to the conservation of impulse occurs when a bubble in translatory motion collapses. In his lively presentation Chincholle gave a survey of the various effects to which this conservation of impulse may give rise. Among them is the well-known phenomenon of a microjet issuing from a collapsing cavitation bubble. Many exciting experiments can be envisaged. For example a small gas bubble, in a liquid which is warmer far below the free surface than at the free surface, will collapse when rising to the colder free surface. As a result a microjet is formed into the ambient atmosphere when the bubble pierces the free surface.

Lewin* (Tekniske Højskole, Copenhagen) talked about bubble dynamics in biological systems. Since there is increasing use of ultrasonic diagnosis, as in obstetrics or in tumour determination, it is of importance to investigate whether this could have any harmful effect. One such an effect might be due to shear stresses caused by vibrating bubbles in biological tissue. Lewin reported on the work in progress: a mathematical model is developed based on Stokes boundary layers and associated acoustic streaming. Bubbles, supposed to be present in the tissue, are excited by the pressure variations in the flow. Experiments to see whether degradation of tissue may occur are carried out with mouse liver tissue under various conditions. This interesting research has not progressed far enough for an opinion to be given about possible harm done by ultrasonic vibrations.

Thomas* (University of Cambridge) gave an outline of the various fluid mechanical phenomena occurring when air is entrained by a water flow. One can think in this connexion of a flow of cooling water over a weir or channel flow in a pool. A problem which arises here and which is studied at Cambridge is: what is the criterion for air entry? The connexion with the Colloquium is that air is entrained mainly as bubbles of various sizes. Thomas showed a film of an air-entraining channel flow (as a Cambridge touch an old shoe appeared as yard stick on the film), on which interesting phenomena could be observed such as the clustering of bubbles in certain parts of the flow. Phenomena like this and many others coming up during the Colloquium are very poorly understood and investigated as yet. It is very helpful for those involved to discuss these problems amidst people coming from various disciplines and fields of experience. We feel that in this sense the Colloquium has been informative and useful for the participants.

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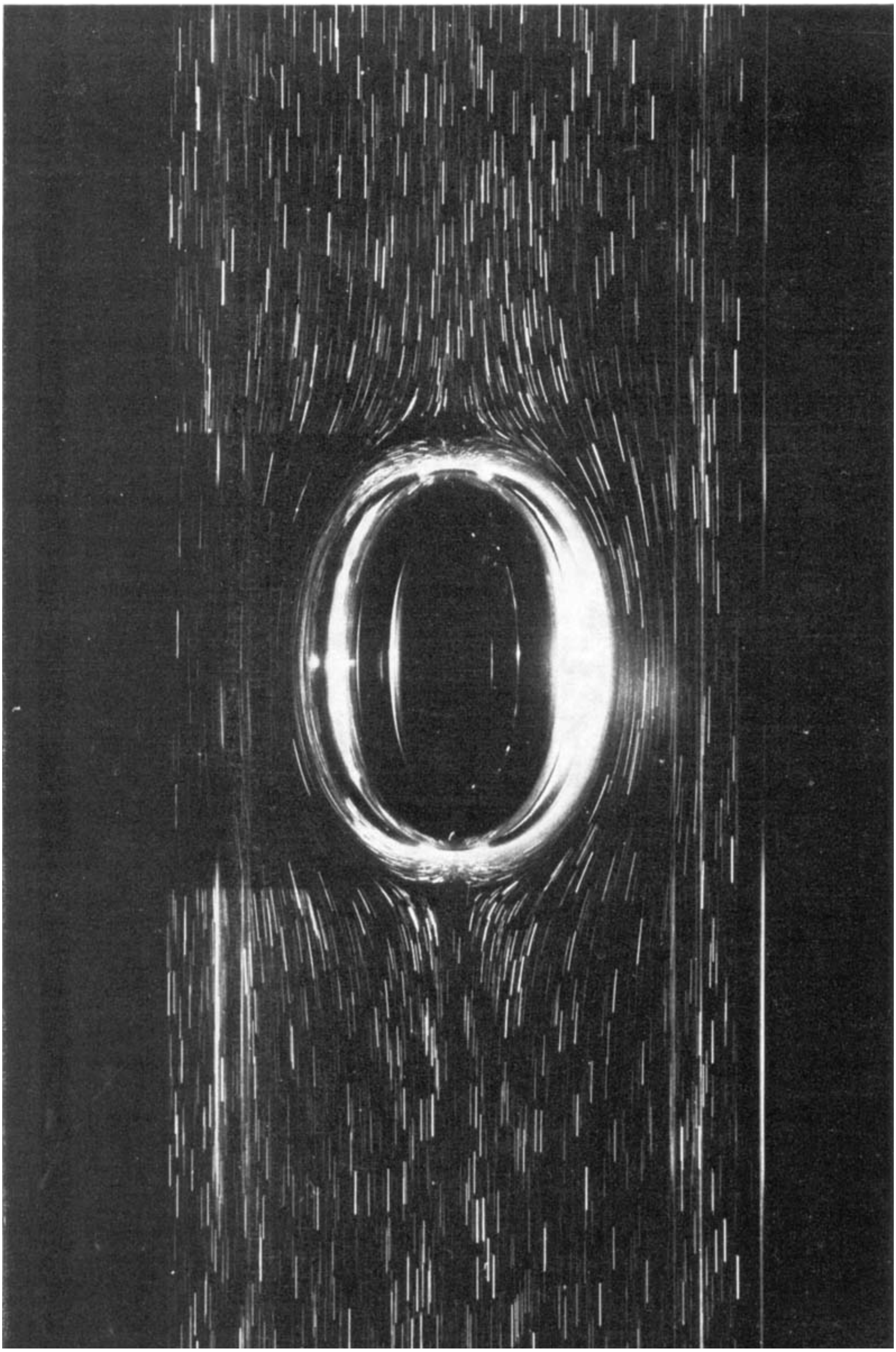

FIGURE 2. Bubble rising in Newtonian liquid. $\lambda = 0{\cdot}65$. Bubble volume $9{\cdot}7 \times 10^{-6}$ m^3.

FIGURE 3. Bubble rising in non-Newtonian liquid. $\lambda = 0{\cdot}32$. Bubble volume $13 \times 10^{-6}\mathrm{m}^3$. (Courtesy of Dr P. Thizon.)